Qubit's Equation of motion

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Outline

- Background
  - quantum state
  - density matrix
- Equation of motion
  - von Neumann equation
  - Lindblad master equation

Quantum state

- Pure state(純態):
  - ■ wavefunction的狀態
  - ■疊加態亦屬於純態

態向量表示法= |ψ⟩

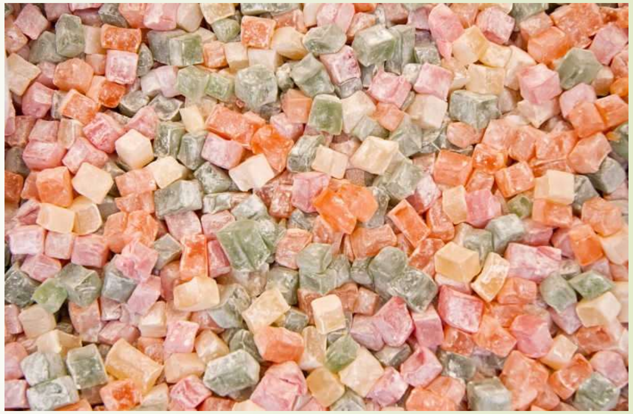
- Mixed state(混合態):
  - ■多種純態依機率組成

態向量表示法=|ψ⟩⟨ψ|

Example(一堆長一樣軟糖)

state:各種純態
(也有可能是apple+banana的疊加態)

state	probability
Apple	20%
Banana	40%
Guava	25%
Strawberry	15%



這一堆軟糖就是一種混和態
➔可以寫成density matrix

Density Matrix(Density operator)

抓出機率的operator
作用在ket上，得到的結果是ket
出現在該系統的各個機率

Define $\hat{\rho} \equiv |\psi\rangle\langle\psi|$

Mixed state: $|\psi\rangle = \sum_m c_m |m\rangle$

1 qubit : $|0\rangle$、 $|1\rangle$
2 qubit: $|00\rangle$、 $|01\rangle$、 $|10\rangle$、

Get $\hat{\rho} = \sum_{m,n} c_m c_n^* |m\rangle\langle n| = \sum_{m,n} \rho_{mn} |m\rangle\langle n|$

其中ρ_{mn}=$c_m c_n^*$

其中$P_m = |c_m| X |c_m|$

P=系統在 |m > 態出現的機率

m=實數

c_m=複數

c_m是複數，可能有phase
所以令其為實數乘上e
φ_m是random phase

Let c_m=$\sqrt{P_m} e^{i\varphi_m}$

$\rho_{mn}=\overline{c_m c_n^*} = \sqrt{P_m P_n e^{i(\varphi_m-\varphi_n)}} = P_m \delta_{mn}$

δ_{mn}中當m=n時則為1

m≠n時則為0

Average後的結果 $\hat{\rho}=\sum_m P_m |m\rangle\langle m|$

這告訴我們:當只有1 qubit時，不會存在$|0\rangle\langle 1|$ 、 $|1\rangle\langle 0|$

只會有$|0\rangle\langle 0|$ 、 $|1\rangle\langle 1|$兩種混合態

Characteristic

- density matrix內的元素必>0
- density matrix內元素總和=1，trace(密度矩陣)=1
- density matrix是Hermitian matrix

$|\psi\rangle\langle\psi|^{+} = |\psi\rangle\langle\psi|$ 是Hermitian matrix
且機率P為實數
因此相乘亦是Hermitian matrix

- 純態的density matrix對角線上只會有一個是1
- 混合態的density matrix對角線上有多個數值且和為1

von Neumann equation

- Equation(在一個封閉的qubit system)

$$\frac{\mathrm{d}\hat{p}}{\mathrm{dt}} = \frac{1}{i\hbar}[\hat{H}, \hat{p}]$$

$\hat{H}$=系統的Hamiltonian
$\hat{p}$=density matrix

- Physical meaning

Qubit在各種operation下，如何演變
Schrodinger equation→純態隨t的演化
von Neumann equation→$\hat{p}$隨t的演化

derivation

Time –dependent Schrodinger equation

$i\hbar \frac{\boldsymbol{d}}{\boldsymbol{dt}} |\boldsymbol{\psi}(\boldsymbol{t})\rangle = \widehat{\boldsymbol{H}}|\boldsymbol{\psi}(\boldsymbol{t})\rangle$

$\hat{\rho}(t)$=$\sum_m P_m \mid \psi_m(t)\rangle\langle\psi_m(\mathrm{t}) \mid$ 帶入上式

得到$i\hbar \frac{\boldsymbol{d}}{\boldsymbol{dt}} \hat{\rho}(t)$=$\sum_m P_m(\widehat{H} \mid \psi_m(\mathrm{t})\rangle\langle\psi_m(\mathrm{t}) \mid - \mid \psi_m(\mathrm{t})\rangle\langle\psi_m(\mathrm{t}) \mid \widehat{H})$

➔ $i\hbar \frac{\boldsymbol{d}}{\boldsymbol{dt}} \hat{\rho}(t)$=$[\widehat{H}, \hat{p}]$

移項

➔ $\frac{\mathrm{d}\hat{p}}{\mathrm{dt}} = \frac{1}{i\hbar}[\widehat{H}, \hat{p}]$

其解

➔ $\hat{\rho}(t)$= $e^{\frac{-i\widehat{H}t}{\hbar}}\ \hat{\rho}(0)\ e^{\frac{i\widehat{H}t}{\hbar}}$

Lindblad master equation

- Equation

$$\frac{d\hat{p}}{dt} = \frac{1}{i\hbar}[\hat{H},\hat{p}] + \sum_k (\widehat{L_k}\,\hat{p}\widehat{L_k}^{+} - \frac{1}{2}\{\widehat{L_k}^{+}\widehat{L_k},\hat{p}\})$$

$\widehat{L_k}$=Lindblad or jump operator
單位=$[s^{-\frac{1}{2}}]$
根據model，選擇$\widehat{L_k}$
Ex: readout resonator(讀取共振器)的$\widehat{L_k}=\sqrt{\frac{\kappa}{2\pi}}\hat{a}$

- Assumption

■Born assumption→qubit和environment的交互作用很小，可以忽略
■Markovian approximation→system的noise process 是memoryless
■qubit's system的初始狀態不與environment糾纏(獨立的)

➔ $\hat{p}$ (t=0)=$\hat{p}_{sys} \times \hat{p}_{env}$

實際上qubit's system是一個開放系統
總會和environment、readout resonator產生interaction

將Lindblad master equation
理論修正➔Input-output theory

Input=驅動system的場
Output= system傳出的場

thank you for your attention

Dispersive Readout

What Dispersive Readout is

detectingthequbitstatebyob-servingtheshift
intheresonancefrequencyofareadoutreson
atorinteractingwiththequbit

Dispersive limit

- We can not detect or control the qubit when $w_r = w_q$ 12
- Detune w_q , $\Delta_{qr} \equiv w_r - w_q$
-

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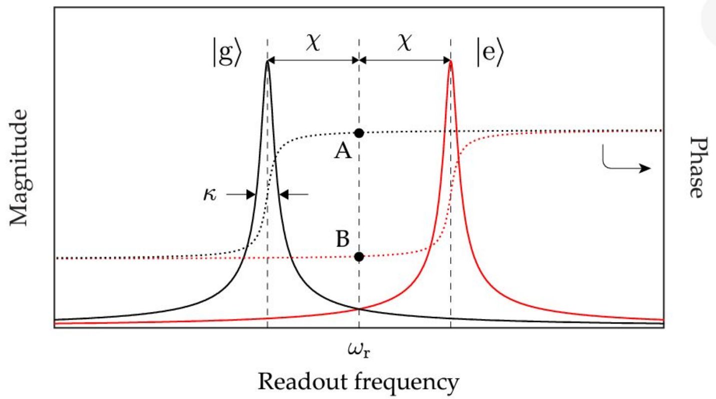
(a) **Shift in resonator frequency: dispersive shift**



$$\hat{\mathcal{H}}_{\mathrm{JC}}^{\mathrm{disp}} \equiv \hat{U}_{\mathrm{disp}} \hat{\mathcal{H}}_{\mathrm{JC}} \hat{U}_{\mathrm{disp}}^{\dagger}$$
$$\approx \hbar(\omega_{\mathrm{q}} + \chi)\frac{\hat{\sigma}_z}{2} + \hbar(\omega_{\mathrm{r}} + \chi\hat{\sigma}_z)\hat{a}^{\dagger}\hat{a},$$

NonDemolition Measurement

- Dispersive term conmutes with bare qubit term and bare resonator term
- the measurement qubit remains in the eigenstate that recorded as a measurement outcome

Ehance Signal-to-Noise Ratio

- increasing the average number of photons ñ for detection
- ñ more less than n_{crit}
- critical photon numbers is given by $\Delta_{qr}/4g^2$

Ensure Fast Readout with High Fedility

- K too large
- K too smal
- best SNR: 2X=K
- $X \equiv g^{2/}\Delta_{qr}$
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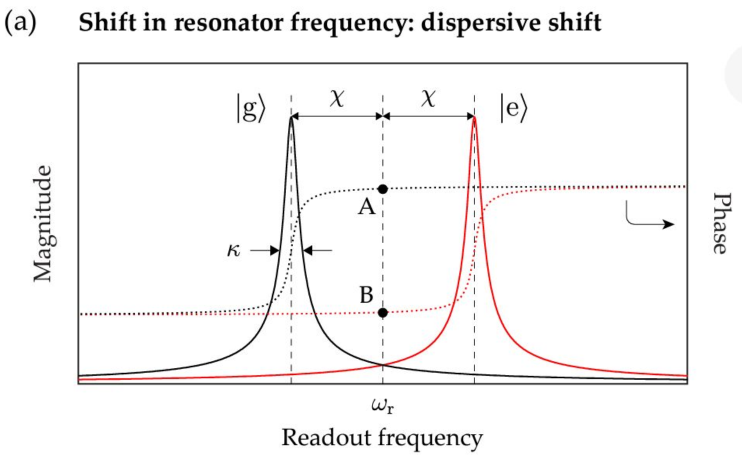

Purcell effect

- cause reduction in T_1
- Δ_{qr} =0 cause the maximine of the effect
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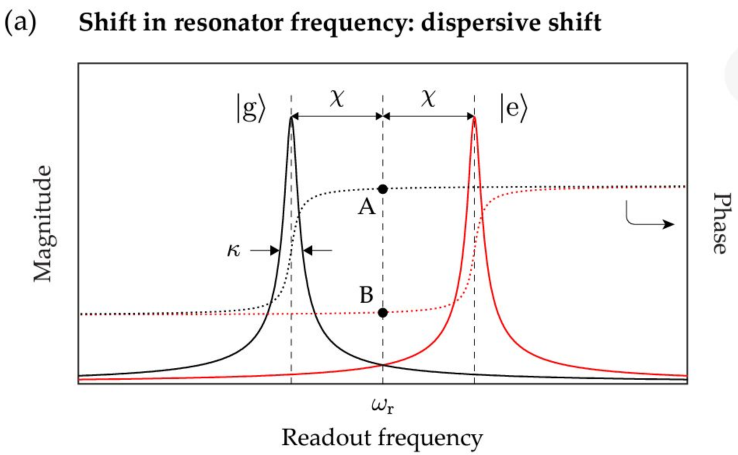


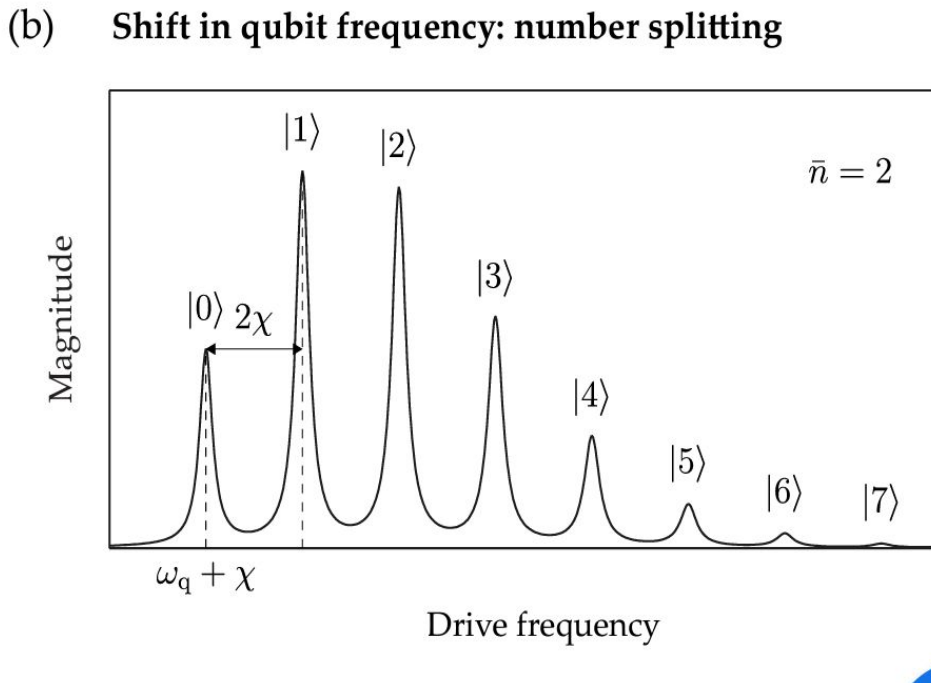
in Real Expirement

- Total Shift:

  [χ01−χ10+Mj=2(χj1−χ1j− χj0 +χ0j)]/2

- Splitting of qubit spectrum

$$\hat{\mathcal{H}}_{\mathrm{JC}}^{\mathrm{disp}} \approx \hbar[\omega_{\mathrm{q}} + \chi(1 + 2\hat{a}^{\dagger}\hat{a})]\frac{\hat{\sigma}_z}{2} + \hbar\omega_{\mathrm{r}}\hat{a}^{\dagger}\hat{a}.$$



Conclution

A set of photons, the energy of each of which is about ω_r, enter the resonator.

The qubit state information is encoded to the photons, for example, as the phase of the transmission, by the qubit-resonator interaction. The measurement-induced dephasing caused by the same interaction makes the qubit state lose its phase coherence and collapse into |0or |1.

The photons escape from the resonator and are then detected.